IFT-P.069/98, ADP-98-66/T333

Fock terms in the quark-meson coupling model

G. Krein[1,2], A.W. Thomas[1] and K. Tsushima[1]

[1]Department of Physics and Mathematical Physics and Special Research Center for the Subatomic Structure of Matter, University of Adelaide, SA 5005, Australia

[2] Instituto de Física Teórica, Universidade Estadual Paulista

Rua Pamplona, 145 - 01405-900 São Paulo, SP, Brazil

Abstract

The mean field description of nuclear matter in the quark-meson coupling model is improved by the inclusion of exchange contributions (Fock terms). The inclusion of Fock terms allows us to explore the momentum dependence of meson-nucleon vertices and the role of pionic degrees of freedom in matter. It is found that the Fock terms maintain the previous predictions of the model for the in-medium properties of the nucleon and for the nuclear incompressibility. The Fock terms significantly increase the absolute values of the single-particle, four-component scalar and vector potentials, a feature that is relevant for the spin-orbit splitting in finite nuclei.

PACS NUMBERS: 21.65.+f, 24.85.+p, 24.10.Jv, 12.39.-x

KEYWORDS: Quark-meson coupling model, bag model, nuclear matter, Fock terms

1. Introduction

The quark-meson coupling (QMC) model [1] provides a simple extension of relativistic many-body models based on point-like hadrons, such as quantum hadrodynamics (QHD) [2], to include the explicit quark structure of the hadrons. Although both QMC and QHD and their variations share many common features, there exist prominent differences. Perhaps the most striking one is the mechanism through which the effective masses of the light vector mesons (ρ, ω and ϕ) decrease in medium. While in QMC the decrease results from an increase in the lower component of the Dirac spinor of the quarks in the mesons [3], in QHD the decrease is driven by the vacuum polarization in medium [4]. Modern versions of QHD, based on the ideas of effective field theory incorporate vacuum polarization effects implicitly through phenomenological effective couplings [5]. However, one can obtain either an increase or a decrease of the masses of the vector mesons when different sets of effective couplings are used – even though they fit ground-state observables of nuclei equally well.

The QMC model has been applied to a great variety of problems in nuclear physics using the mean field approximation [6]. In this paper we extend the model to include the exchange, or Fock terms, which are required by the Pauli exclusion principle. As in previous applications of QMC, we consider nuclear matter as a system of non-overlapping bags and effects due to quark-exchange between different bags [7] are neglected. Thus the Pauli principle is enforced at the nucleon level. It is important to consider Fock terms not only as a matter of principle, but also because it is only through them that the momentum dependence of the meson-nucleon vertices can be explored. As we know, for example, from the work of Bouyssy et al. [8] within QHD, the momentum dependence of the exchange contributions from the isovector mesons (π and ρ) is essential to explain the magnitude and the systematic behaviour with N and Z of the spin-orbit splittings in finite nuclei.

In the next section we formulate the QMC model in the language of an effective meson-nucleon Hamiltonian following the ideas of the Cloudy Bag Model (CBM) [9]. In Section 3 we obtain the expressions for the nuclear matter energy density and of the single-particle Fock potential. The numerical results are presented and discussed in Section 4. In Section 5 we present our conclusions and discuss perspectives for future calculations.

2. Effective meson-nucleon Hamiltonian

The Lagrangian density of the model is

$$\begin{aligned}\mathcal{L}_{\rm QMC} = \mathcal{L}_{\rm MIT} &+ \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2\,\omega_\mu\omega^\mu \\ &+ \frac{1}{2}\left(\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} - m_\pi^2\boldsymbol{\pi}^2\right) - \frac{1}{4}\boldsymbol{G}_{\mu\nu}\cdot\boldsymbol{G}^{\mu\nu} + \frac{1}{2}m_\rho^2\,\boldsymbol{b}_\mu\cdot\boldsymbol{b}^\mu \\ &+ \bar{\psi}\left(g_\sigma^q\sigma - g_\omega^q\gamma_\mu\omega^\mu - \frac{1}{2f_\pi}\gamma_\mu\gamma_5\boldsymbol{\tau}\cdot\partial_\mu\boldsymbol{\pi} - g_\rho^q\frac{1}{2}\gamma_\mu\boldsymbol{\tau}\cdot\boldsymbol{b}^\mu\right)\psi\,\theta_V \end{aligned} \tag{1}$$

where $\mathcal{L}_{\rm MIT}$ is the Lagrangian density of the MIT Bag Model,

$$\mathcal{L}_{\rm MIT} = \left[\bar{\psi}\left(i\gamma_\mu\partial^\mu - m_q\right)\psi - B\right]\theta_V - \frac{1}{2}\bar{\psi}\psi\,\delta_S, \tag{2}$$

θ_V is one inside the bag and zero outside, δ_S is a delta function on the bag surface, $F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$, and $\boldsymbol{G}_{\mu\nu} = \partial_\mu\boldsymbol{b}_\nu - \partial_\nu\boldsymbol{b}_\mu$. The σ and ω^μ field operators have constant expectation values in symmetric nuclear matter, while $\boldsymbol{b}^\mu$ field has a nonzero expectation value only in asymmetric nuclear matter, and the $\boldsymbol{\pi}$ field has zero expectation value in both symmetric and asymmetric matter, because of parity considerations. We then separate from the meson field operators their mean field values and treat the fluctuations in time-ordered perturbation theory. More specifically, we begin by solving the single-nucleon problem in the presence of the constant mean-fields. Then we project the quark-meson Hamiltonian, obtained from the fluctuating meson fields coupled to the quarks, onto the space of the single-nucleon states in the presence of the constant mean meson fields.

The Hamiltonian resulting from this procedure is similar to the usual CBM Hamiltonian, with the difference that the effective meson-nucleon vertices in the present case are evaluated with density dependent wave functions. In this approach exchange effects from quarks in different nucleons are not taken into account. We also note that our calculation is not fully self-consistent, in the sense that the Fock terms are calculated perturbatively using Hartree self-energies. Our approach is similar to Chin's in QHD [10]. There, the difference between the perturbative and full, self-consistent results was not very large, because the Hartree terms provide by far the most important contributions. Since the Hartree terms also dominate in QMC, we expect that such a perturbative approach should be sufficiently reliable for a first estimate of the Fock terms in this case, and reserve for a future work a more complete, self-consistent calculation.

Let σ_0, ω_0 and b_0 denote the meson mean fields. Let $B_\lambda^\dagger(\boldsymbol{p})$ and $B_\lambda(\boldsymbol{p})$ denote the creation and annihilation operators for single-nucleon states with spin-isospin λ and momentum $\boldsymbol{p}$ in the presence of the mean fields. That is, the operator $B_\lambda^\dagger(\boldsymbol{p})$ creates the state

$$|\boldsymbol{p},\lambda\rangle = B^\dagger_\lambda(\boldsymbol{p})|0\rangle, \tag{3}$$

which has energy

$$\varepsilon(\boldsymbol{p},\lambda) = E^*(\boldsymbol{p}) + 3g^q_\omega\,\omega_0 + \frac{1}{2}g^q_\rho\,\langle\lambda|\tau_3|\lambda\rangle\,b_0\,, \tag{4}$$

with $E^*(\boldsymbol{p}) = \sqrt{\boldsymbol{p}^2 + M^{*2}}$. In the present paper we adopt the nonrelativistic normalization,

$$\langle\boldsymbol{p}',\lambda'|\boldsymbol{p},\lambda\rangle = \delta(\boldsymbol{p}'-\boldsymbol{p})\,\delta_{\lambda'\lambda}. \tag{5}$$

The effective nucleon mass, M^*, is obtained by solving the MIT bag equations in the presence of the mean fields. It is given explicitly by [11]

$$M^* = \frac{3\Omega - z_0}{R^*} + \frac{4}{3}\pi B R^{*3}, \tag{6}$$

where we have taken equal up and down quark masses, z_0 is the parameter that takes into account zero-point and c.m. motion and

$$\Omega = \sqrt{x^{*2} + (R^* m^*_q)^2}, \tag{7}$$

with

$$m^*_q = m_q - g^q_\sigma \sigma_0. \tag{8}$$

Here, x^* is the solution of the transcendental equation resulting from the linear (confining) boundary condition at the bag surface:

$$j_0(x^*) = \sqrt{\frac{\Omega - R^* m^*_q}{\Omega + R^* m^*_q}}\; j_1(x^*), \tag{9}$$

where j_0 and and j_1 are spherical Bessel functions. The in-medium radius, R^*, is obtained by minimizing M^* with respect to R^*.

The next step is to project the quark-meson Hamiltonian, obtained from the Lagrangian density given in Eq. (1), onto the single-nucleon states of Eq. (3). The resulting effective meson-nucleon Hamiltonian can be written as

$$H_{\rm eff} = H_{\rm MMF} + H_0 + W, \tag{10}$$

where $H_{\rm MMF}$ is the Hamiltonian of the meson mean fields,

$$H_{\rm MMF} = \frac{1}{2}m^2_\sigma \sigma^2_0 - \frac{1}{2}m^2_\omega\,\omega^2_0 - \frac{1}{2}m^2_\rho\,b^2_0, \tag{11}$$

H_0 is the sum of the single-nucleon and single-meson Hamiltonians, and W is the meson-nucleon interaction. Specifically, H_0 is the sum

$$H_0 = \sum_\lambda \int d\boldsymbol{p}\, \varepsilon(\boldsymbol{p}, \lambda)\, B^\dagger_\lambda(\boldsymbol{p})\, B_\lambda(\boldsymbol{p}) + \sum_{j=\sigma,\omega,\pi,\rho} \int d\boldsymbol{k}\, \omega_j(\boldsymbol{k})\, a^\dagger_j(\boldsymbol{k})\, a_j(\boldsymbol{k}), \tag{12}$$

where $\varepsilon(\boldsymbol{p}, \lambda)$ is given in Eq. (4), $\omega_j(\boldsymbol{k}) = (\boldsymbol{k}^2 + m_j^2)^{1/2}$, and $a^\dagger_j(\boldsymbol{k})$ and $a_j(\boldsymbol{k})$ are the meson creation and annihilation operators. The meson-nucleon interaction, W, can be written as

$$W = \sum_{j=\sigma,\omega,\pi,\rho} \int d\boldsymbol{p}\, d\boldsymbol{p}'\, d\boldsymbol{k}\, \delta(\boldsymbol{k} - \boldsymbol{p}' + \boldsymbol{p})\, \frac{(2\pi)^{3/2}}{\sqrt{2\omega_j(\boldsymbol{k})}} \sum_{\lambda\lambda'} W^j_{\lambda'\lambda}(\boldsymbol{p}', \boldsymbol{p})\, B^\dagger_{\lambda'}(\boldsymbol{p}')\, B_\lambda(\boldsymbol{p})\, a_j(\boldsymbol{k}) + \text{h.c.}, \tag{13}$$

where the vertices $W^j_{\lambda'\lambda}(\boldsymbol{p}', \boldsymbol{p})$ are of the general form

$$W^j_{\lambda'\lambda}(\boldsymbol{p}', \boldsymbol{p}) = \langle \boldsymbol{p}'\lambda' | \bar{\psi}(0)\Gamma^j\psi(0) | \boldsymbol{p}\lambda \rangle, \tag{14}$$

with

$$\Gamma^\sigma = -g^q_\sigma, \tag{15}$$

$$\Gamma^\omega = -g^q_\omega \not{\epsilon}_\omega, \tag{16}$$

$$\Gamma^\pi = i\frac{f^q_\pi}{m_\pi}\gamma^\mu(p' - p)_\mu\gamma_5\boldsymbol{\tau}, \tag{17}$$

$$\Gamma^\rho = -\frac{1}{2}g^q_\rho \not{\epsilon}_\rho\, \boldsymbol{\tau}. \tag{18}$$

Here $k = (E^*(\boldsymbol{p}') - E^*(\boldsymbol{p}), \boldsymbol{p}' - \boldsymbol{p})$, and the polarization vectors $\epsilon^\mu(\boldsymbol{k})$ satisfy

$$\sum_{s=1}^{3} \epsilon^\mu_{\omega,\rho}(\boldsymbol{k}, s)\epsilon^\nu_{\omega,\rho}(\boldsymbol{k}, s) = -g^{\mu\nu} + \frac{k^\mu k^\nu}{m^2_{\omega,\rho}}. \tag{19}$$

The next step requires evaluation of the matrix elements $W^j_{\lambda'\lambda}(\boldsymbol{p}', \boldsymbol{p})$. For free-space nucleon states, matrix elements of this sort have been calculated in the MIT bag model in a variety of approximations. In the present case, the situation is more complicated because the states $|\boldsymbol{p}, \lambda\rangle$ have their mass-shell modified in medium. Not only is the nucleon mass modified from its free-space value, M, to M^*, but the relation between energy and momentum is also modified by the mean vector fields, as can be seen in Eq. (4). These modifications are, of course, the interesting aspect of the QMC model as they will introduce a density dependence into the form factors. In order not to complicate the problem too much, in this initial study we adopt a simple approach. First, we parametrize the various matrix elements $W^j_{\lambda'\lambda}(\boldsymbol{p}', \boldsymbol{p})$ as in free space, namely

$$W^{j}_{\lambda'\lambda}(\boldsymbol{p}', \boldsymbol{p}) = \bar{u}(\boldsymbol{p}', \lambda')\, O^{j}(k)\, u(\boldsymbol{p}, \lambda), \tag{20}$$

with

$$O^{\sigma}(k) = -g^{q}_{\sigma}\, F_{s}(k^{2}) \tag{21}$$

$$O^{\omega}(k) = -g^{q}_{\omega}\, \epsilon^{\mu}_{\omega} \left[\gamma_{\mu} F^{(\omega)}_{1}(k^{2}) + \frac{i\sigma_{\mu\nu}k^{\nu}}{2M^{*}} F^{(\omega)}_{2}(k^{2})\right], \tag{22}$$

$$O^{\pi}(k) = i\frac{1}{2f_{\pi}}\, G_{A}(k^{2})\ \not{k}\gamma_{5}\boldsymbol{\tau}, \tag{23}$$

$$O^{\rho}(k) = -\frac{1}{2} g^{q}_{\rho}\, \epsilon^{\mu}_{\rho} \left[\gamma_{\mu} F^{(\rho)}_{1}(k^{2}) + \frac{i\sigma_{\mu\nu}k^{\nu}}{2M^{*}} F^{(\rho)}_{2}(k^{2})\right] \boldsymbol{\tau}. \tag{24}$$

The Dirac spinors, $u(\boldsymbol{p}, \lambda)$, are given by

$$u(\boldsymbol{p}, \lambda) = \sqrt{\frac{E^{*}(\boldsymbol{p}) + M^{*}}{2E^{*}(\boldsymbol{p})}} \begin{pmatrix} 1 \\ \dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E^{*}(\boldsymbol{p}) + M^{*}} \end{pmatrix} \chi_{s}\xi_{t}, \tag{25}$$

where χ_{s} and ξ_{t} are the Pauli spinors for spin and isospin, and $E^{*}(\boldsymbol{p}) = \sqrt{\boldsymbol{p}^{2} + M^{*2}}$. Next, we calculate the various form factors in the usual way, using, however, the bag model wave-functions modified by the mean fields. In calculating the form factors we ignore center-of-mass and recoil corrections, as well as effects due to Lorentz contraction when going to the Breit frame [12]. Both approximations can be improved by using the technique recently developed in Ref. [13], but no qualitative changes with respect to their density dependence are expected in using the present approximations. We note that in Eq. (23) we have not included the induced pseudoscalar form factor; we intend to investigate its effect on nuclear matter properties when using a chiral model, such as the CBM.

Let the single-quark wave functions in the presence of the mean fields be written as

$$q(\boldsymbol{r}) = \begin{pmatrix} g(r) \\ i\boldsymbol{\sigma}\cdot\hat{\boldsymbol{r}} f(r) \end{pmatrix} \phi(\hat{\boldsymbol{r}}), \tag{26}$$

where $\phi(\hat{\boldsymbol{r}})$ contains the spin-isospin wave functions. In calculating the vector meson form factors, we follow the standard procedure of relating the Dirac form factors F_{1} and F_{2} to the Sachs form factors, G_{E} and G_{M}, as $F_{1} = [G_{E} + \eta G_{M}]/(1+\eta)$ and $F_{2} = [G_{M} - G_{E}]/(1+\eta)$ with $\eta = -k^{2}/4M^{*2}$. The various form factors in Eqs. (21) - (24) are given by:

$$F_{s}(k^{2}) = 3\int d\boldsymbol{r}\, j_{0}(kr) \left[g^{2}(r) - f^{2}(r)\right], \tag{27}$$

$$G^{\omega}_{E}(k^{2}) = 3\int d\boldsymbol{r}\, j_{0}(kr) \left[g^{2}(r) + f^{2}(r)\right] \equiv 3\, G_{E}(k^{2}), \tag{28}$$

$$G_M^\omega(k^2) = 2M^* \int d\boldsymbol{r}\, \frac{j_1(kr)}{k}\, [2g(r)f(r)] \equiv G_M(k^2), \tag{29}$$

$$G_A(k^2) = \frac{5}{3} \int d\boldsymbol{r}\, j_0(kr) \left[g^2(r) - \frac{1}{3} f^2(r) \right], \tag{30}$$

$$G_E^\rho(k^2) = \frac{1}{3}\, G_E^\omega(k^2) = G_E(k^2), \tag{31}$$

$$G_M^\rho(k^2) = \frac{5}{3}\, G_M^\omega(k^2) = \frac{5}{3}\, G_M(k^2). \tag{32}$$

Note that G_E and G_M (without meson indices) are the usual, electromagnetic Sachs form factors.

3. Nuclear matter energy and the Fock potential.

The energy density of symmetric nuclear matter (in which case the mean field b_0 does not contribute) is given by the sum of the mean-field energies and the Fock energy,

$$\mathcal{E}_{NM} = 4 \int \frac{d\boldsymbol{p}}{(2\pi)^3}\, \theta(k_F - |\boldsymbol{p}|)\, E^*(\boldsymbol{p}) \; + 3\, g_\omega^q \omega_0\, \rho_N + \frac{1}{2} m_\sigma^2 \sigma_0^2 - \frac{1}{2} m_\omega^2 \omega_0^2 + \sum_{j=\sigma,\omega,\pi,\rho} \mathcal{E}^j_{\text{Fock}}, \tag{33}$$

where ρ_N is the nucleon density and the Fock energies $\mathcal{E}^j_{\text{Fock}}, j = \sigma, \omega, \pi, \rho$ are the second-order exchange contributions

$$\mathcal{E}^j_{\text{Fock}} = \frac{1}{2} \int \frac{d\boldsymbol{p}}{(2\pi)^3}\, \theta(k_F - |\boldsymbol{p}|) \int \frac{d\boldsymbol{p}'}{(2\pi)^3}\, \theta(k_F - |\boldsymbol{p}'|)\, \frac{\sum_{\lambda\lambda'} |W^j_{\lambda\lambda'}(\boldsymbol{p}, \boldsymbol{p}')|^2}{(\boldsymbol{p}' - \boldsymbol{p})^2 + m_j^2}. \tag{34}$$

Using Eqs. (21)-(24) in the expression for the Fock energy density, Eq. (34), we can rewrite it as

$$\mathcal{E}^j_{\text{Fock}} = \frac{1}{2} \int \frac{d\boldsymbol{p}}{(2\pi)^3}\, \theta(k_F - |\boldsymbol{p}|) \sum_\lambda \bar{u}(\boldsymbol{p}, \lambda)\, U^j_{\text{Fock}}(\boldsymbol{p}) u(\boldsymbol{p}, \lambda) \tag{35}$$

with the "Fock potential", $U^j_{\text{Fock}}(\boldsymbol{p})$, or the exchange nucleon self-energy given by

$$U^j_{\text{Fock}} = \int \frac{d\boldsymbol{p}}{(2\pi)^3}\, \theta(k_F - |\boldsymbol{p}|) O^j(-k) \sum_{\lambda'} u(\boldsymbol{p}', \lambda') \bar{u}(\boldsymbol{p}', \lambda')\, O^j(k), \tag{36}$$

where the four-momentum k is given as before. The potential $U^j_{\text{Fock}}(\boldsymbol{p})$ for each meson has the Dirac structure,

$$U^j_{\text{Fock}}(\boldsymbol{p}) = U^j_s(\boldsymbol{p}) + \gamma^0\, U^j_0(\boldsymbol{p}) + \boldsymbol{\gamma}\cdot\hat{\boldsymbol{p}}\, U^j_v. \tag{37}$$

The explicit forms of the potentials are easily obtained by making use of Eqs. (21)-(24) and will be given in a separate publication.

The final step is to determine the scalar mean field, σ_0, which satisfies the self-consistency equation

$$\sigma_0 = \frac{g_\sigma}{m_\sigma^2} C(\sigma_0)\, 4 \int \frac{d\boldsymbol{p}}{(2\pi)^3}\, \theta(k_F - |\boldsymbol{p}|) \frac{M^*}{E^*(\boldsymbol{p})} + \frac{1}{m_\sigma^2} \frac{\partial}{\partial \sigma_0} \sum_{j=\sigma,\omega,\pi,\rho} \mathcal{E}^j_{\rm Fock}, \tag{38}$$

where

$$g_\sigma = 3 g_\sigma^q S(0), \qquad\qquad C(\sigma_0) = \frac{S(\sigma_0)}{S(0)}, \tag{39}$$

with

$$S(\sigma_0) = \frac{\Omega/2 + m_q^* R^* (\Omega - 1)}{\Omega(\Omega - 1) + m_q^* R^*/2}. \tag{40}$$

The mean vector field, ω_0, is given in terms of the baryon density, as usual.

4. Numerical results

Before presenting the numerical results, we discuss some technical points. As is well known, the sharp surface of the bag induces oscillations in the form factors at large momenta. The process of removing spurious centre of mass motion and projecting onto a definite momentum tends to smooth this, but it is extremely time consuming. In order to avoid this time consuming calculation in this first investigation, we have chosen to parametrize the quark wave functions in terms of smooth functions – following Ref. [14] we use a gaussian form with two parameters (R_0 and β). These are adjusted to reproduce the r.m.s. radius and the quark scalar density of the nucleon calculated with the original MIT bag wave functions. Note that R_0 and β are density dependent.

We also observe that when c.m. corrections and the meson cloud are neglected in the calculation of form factors, some physical quantities such as g_A and the magnetic moments are not well reproduced within the model. For example, $g_A = G_A(0) \sim 1.09$ for the "bare" bag without c.m. corrections, whereas the experimental value is $g_A = 1.267$ [16]. The magnetic moments are given in the tensor couplings of the ω and ρ,

$$\begin{aligned} \kappa^S &= \mu_p + \mu_n - 1 = \frac{g_\omega^q F_2^\omega(0)}{g_\omega^q F_1^\omega(0)} = -1 + \frac{1}{3}\, G_M(0), \\ \kappa^V &= \mu_p - \mu_n - 1 = \frac{g_\rho^q F_2^\rho(0)}{g_\rho^q F_1^\rho(0)} = -1 + \frac{5}{3}\, G_M(0). \end{aligned} \tag{41}$$

Pionic corrections are known [15] to be very important for these quantities. For the nuclear matter calculation, the main effect of neglecting these is to underestimate the tensor coupling

of the ρ to the nucleon, κ^V (κ^S is a small quantity and does not play a big role for the binding energy of nuclear matter). Therefore, in order to obtain a more realistic estimate of the contribution of the tensor coupling of the ρ we adjust $\kappa^V = \mu_p - \mu_n - 1$ to its experimental value. We also adjust $G_A(0)$ to the experimental value of g_A. As we said above, the important aspect for our calculation is the density dependence of the form factors, and we do not expect qualitative changes in a more complete calculation.

We begin by investigating the effect of adding the Fock energy to the mean field energy (Hartree) per nucleon. We use the σ and ω coupling constants of the Hartree approximation to calculate the Fock terms, but solve Eq. (38) self-consistently to obtain σ_0 and M^*. The second term in Eq. (19) does not contribute in the case of ω coupling because of baryon current conservation, and for ρ coupling it gives an extremely small contribution to the nuclear binding energy and is therefore neglected. The Hartree coupling constants are fixed by requiring a stable minimum at $E/A - M = -15.7$ MeV at $\rho_0 = 0.15$ fm^{-3}. For a free bag radius of 0.8 fm they are given [11] by $g_\sigma^2/4\pi = 5.40$ and $g_\omega^2/4\pi = 5.31$, where g_σ is defined in Eq. (39) and $g_\omega = 3g_\omega^q$. We consider first the Fock terms associated with just the σ and ω. In Figure 1 we show the results. The effect of the Fock terms is repulsive and of the order of 5 MeV.

In Figure 2 we show the effect of the pion Fock term. The pion gives a repulsive contribution of the order of 8 MeV. Here, as in QHD, we have the situation that the effective NN interaction due to pion exchange contains a short-range contact interaction. There have been arguments that contact interactions should not be taken into account in a mean-field calculation, since they are suppressed by short-range NN correlations [8]. In a complete calculation, in which the full ladder diagrams are summed, this suppression of short-distance pion exchange is automatic. In a Hartree-Fock treatment, there is no unambiguous way to subtract such short range pieces from a relativistic interaction. We approach the problem by making a static approximation and expanding the nucleon energies in the spinors as

$$E^*(\boldsymbol{p}) \simeq M^* + \boldsymbol{p}^2/2M^*. \tag{42}$$

The contact interaction then becomes evident and can easily be subtracted. Specifically, the contribution of the pion to the energy density of nuclear matter is given by

$$\begin{aligned}\mathcal{E}^\pi = \frac{1}{2} \times 12 \int \frac{d\boldsymbol{p}}{(2\pi)^3}\theta(k_F - |\boldsymbol{p}|) \int \frac{d\boldsymbol{p}'}{(2\pi)^3}\theta(k_F - |\boldsymbol{p}'|)\, \frac{[G_A(p'-p)]^2}{(\boldsymbol{p}'-\boldsymbol{p})^2 + m_\pi^2} \\ \times 2M^{*2}\Big[E^*(\boldsymbol{p})E^*(\boldsymbol{p}') - M^{*2} - \boldsymbol{p}\cdot\boldsymbol{p}'\Big].\end{aligned} \tag{43}$$

Expanding E^* as in Eq. (42), we obtain

$$\begin{aligned}
\mathcal{E}^\pi &\simeq 6 \int \frac{d\boldsymbol{p}}{(2\pi)^3}\theta(k_F - |\boldsymbol{p}|) \int \frac{d\boldsymbol{p}'}{(2\pi)^3}\theta(k_F - |\boldsymbol{p}'|)\,[G_A(\boldsymbol{p}'-\boldsymbol{p})]^2\, \frac{(\boldsymbol{p}'-\boldsymbol{p})^2}{(\boldsymbol{p}'-\boldsymbol{p})^2 + m_\pi^2} \\
&= 6 \int \frac{d\boldsymbol{p}}{(2\pi)^3}\theta(k_F - |\boldsymbol{p}|) \int \frac{d\boldsymbol{p}'}{(2\pi)^3}\theta(k_F - |\boldsymbol{p}'|)\,[G_A(\boldsymbol{p}'-\boldsymbol{p})]^2 \left[1 - \frac{m_\pi^2}{(\boldsymbol{p}'-\boldsymbol{p})^2 + m_\pi^2}\right]. \qquad (44)
\end{aligned}$$

We have recalculated the energy density using the approximate expression, Eq. (44), instead of Eq. (43). We found that the approximate expression induces an extremely small change in the result, indicating that the terms higher than $\mathcal{O}(p^2)$ neglected in the expansion give a negligible contribution to the energy density. The factor 1 in the square brackets in Eq. (44) is due to the contact interaction in the one-pion-exchange. When this factor is subtracted, the contribution of the pion becomes attractive (dotted line in Figure 2), and is of the order of 5 MeV.

The ρ also gives a repulsive contribution. Because of the large value of κ^V, the tensor term gives by far the most important contribution. As in the case of the pion, the tensor component contains a contact term. We proceed as previously, make a static approximation and subtract the contact term. The energy per particle is insensitive to the static approximation. For $g_\rho(= g_\rho^q)$ we use the preferred phenomenological value $g_\rho = 2\sqrt{4\pi \times 0.55}$ and readjust the σ and ω coupling constants such as to refit $E/A - M = -15.7$ MeV at $\rho_0 = 0.15\text{fm}^{-3}$. The new values of the coupling constants for a quark mass of $m_q = 5$ MeV and for three different values for the free bag radius, 0.6 fm, 0.8 fm and 1.0 fm are presented in Table I. The values of the ratios of in-medium to free nucleon masses, M^*/M, and r.m.s. radii, r^*/r, at the saturation density are also shown in Table I. In the last two columns of the table we present the values for the nuclear matter incompressibility and symmetry energy. To calculate the contribution from the Fock terms to the symmetry energy we have used the approximation of Bouyssy et. al, discussed on page 387 of Ref. [8]. Different values for the quark mass between 0 and 10 MeV do not change the qualitative results.

Inspection of Table I reveals that the new values for the in-medium bag parameters have not changed much from their Hartree values [11]. For example, for $R = 0.8$ fm, we find that $M^*/M = 0.83$ and $r^*/r = 1.02$. The corresponding Hartree values are $M^*/M = 0.80$ and $r^*/r = 1.09$. Also the incompressibility of nuclear matter is not changed much by the addition of the Fock terms. From the Hartree value, $K = 280$ MeV, it has changed to $K = 285$ MeV. The symmetry energy, a_4, is almost entirely determined by the value of g_ρ, as is well known. Our value for g_ρ, which has *not* been adjusted to the symmetry energy,

predicts $a_4 \sim 30$ MeV, independently of the free bag radius. The experimental value is close to 33 MeV. The contribution of the Fock terms is substantial, of the order of 10 MeV. In Figure 3 we plot the binding energy as a function of k_F corresponding to parameter set (b) in Table I. For comparison, we also present the Hartree solution in the same figure.

Next we discuss the Fock potentials. It is convenient to introduce the sums $V_s = -\, g_\sigma\, \sigma_0 + U_s$ and $V_0 = g_\omega\, \omega_0 + U_0$, which are the combinations that will appear in the single-nucleon Dirac equation. The momentum dependence of the components V_s, V_0 and U_v at the saturation density are shown in Figure 4. The separate contributions of the different mesons to these are shown in Table II, for the three parameter sets of Table I. The addition of the Fock terms increases both the U_s and U_0 components, and the component U_v is very small, for all three sets of parameters. For parameter set (b) of Table I, we find that $|V_s| + |V_0|$ at $|\boldsymbol{p}| = k_F$ has increased by 67 MeV, in comparison with the value in the Hartree approximation. Such an increase will have important consequences for the spin-orbit splittings in finite nuclei. However, a quantitative estimate of such effects for finite nuclei requires a dedicated calculation, which is complicated by the non-locality of the Fock interaction. We leave this for a future study.

We have also investigated the role of the medium dependence of the form factors on the saturation properties of nuclear matter. As we mentioned previously, the self consistent change of the internal structure of the nucleon with the nuclear medium properties is the attractive new aspect of the QMC model, and it is important to know the role of such effects on our results. In order to test these effects, we re-evaluated the Fock terms using the form factors of free space bags, and redetermined the σ and ω coupling constants so as to obtain $E/A - M = -15.7$ MeV at $\rho_0 = 0.15$ fm^{-3}. For $R = 0.8$ fm, the new values are $g_\sigma^2/4\pi = 4.37$ and $g_\omega^2/4\pi = 5.24$. The ratios M^*/M and r^*/r remain almost the same as in Table I, but the incompressibility and the symmetry energy change substantially. The incompressibility changes from $K = 285$ MeV to $K = 264$ MeV, and the symmetry energy from $a_4 = 30$ MeV to $a_4 = 36$ MeV. These big effects arise from an increase of the Fock term of the σ meson. For example, to the 6 MeV change in a_4, the σ meson alone contributes 5 MeV. The reason for this large effect is simple to understand. The form factor F_s in Eq. (27) involves the difference of the squares of the upper and lower components of the quark wave functions, and this difference is strongly density dependent; the difference decreases as the density increases. This density dependence is at the heart of the saturation mechanism in the QMC model, and here, this effect is very visible.

5. Conclusions and perspectives

In this paper we have developed a calculational scheme to introduce Fock (exchange) terms in the QMC model. Following the ideas of the Cloudy Bag Model, we have constructed an effective meson-nucleon Hamiltonian which naturally incorporates momentum- and density-dependent vertices. We found that the mean-field predictions of the QMC model for the in-medium properties of the nucleons are maintained with the incorporation of the Fock terms. The nuclear matter incompressibility remains low as in the mean field approximation. Without adjusting the coupling constant of the ρ meson, the symmetry energy is predicted to be very close to its experimental value. The Fock terms increase the value of the four-component, single-nucleon scalar and vector potentials, which is a welcome feature for the phenomenology of the level splitting in finite nuclei.

The formulation of the present model can be extended in several ways. We intend to incorporate the effects of chiral symmetry and the $\Delta(1232)$. As with nucleons in free space, we expect the Cloudy Bag Model to provide insight on the role of chiral symmetry on in-medium nucleon and nuclear matter properties. The present formulation of the QMC model seems to be particularly convenient for the implementation of the coupled cluster method for studying nucleon correlations in matter [9]. We also intend to perform a fully self-consistent calculation of the Fock terms. With respect to the spin-orbit splitting in finite nuclei, it might be interesting to investigate the effect of Fock terms within an extended model with a density-dependent bag constant. As shown by Jennings and collaborators [17], when the bag constant is allowed to decrease as a function of the density, the description of the spin-orbit splittings is improved. Finally, it will be interesting to see whether, given the implicit density dependence of masses and coupling constants in QMC, one still needs non-linear meson-meson couplings for a good phenomenological description of finite nuclei.

Acknowledgments.

This work was supported in part by the Australian Research Council and the Brazilian agencies CNPq and FAPESP. G. K. would like to thank the CSSM for the financial support and warm hospitality.

REFERENCES

[1] P.A.M. Guichon, Phys. Lett. B 200 (1988) 235; S. Fleck, W. Bentz, K. Shimizu, and K. Yazaki, Nucl. Phys. A 510 (1990) 731.

[2] J.D. Walecka, Ann. Phys. (N.Y.) 83 (1974) 497; B.D. Serot and J.D. Walecka, Adv. Nucl. Phys. 16 (1986) 1.

[3] K. Saito and A.W. Thomas, Phys. Rev. C 51 (1995) 2757; K. Saito, K. Tsushima and A.W. Thomas, Phys. Rev. C 56 (1997) 566.

[4] H.-C. Jean, J. Piekarewicz and A.G. Williams, Phys. Rev. C 49 (1994) 1981; K. Saito, K. Tsushima, A.W. Thomas and A.G. Williams, Phys. Lett. B 433 (1998) 243.

[5] R. J. Furnstahl, B. D. Serot and H.-B. Tang, Nucl. Phys. A 615 (1997) 441; B. D. Serot and J. D. Walecka , Int.J.Mod.Phys. E 6 (1997) 515.

[6] For a recent list of references see: A.W. Thomas, D.H. Lu, K. Tsushima, A.G. Williams and K. Saito, nucl-th/980727.

[7] M. Bracco, G. Krein and M. Nielsen, Phys. Lett. B 432 (1998) 258 .

[8] A. Bouyssy, J.-F. Mathiot, N. Van Giai and S. Marcus, Phys. Rev. C 36 (1987) 380.

[9] A.W. Thomas, Adv. Nucl. Phys. 13 (1984) 1 ; G.A. Miller, Int. Rev. Nucl. Phys. 2 (1984) 190; S. Théberge *et al.*, Phys. Rev. D 22 (1980) 2838; D 23 (1981) 2106(E).

[10] S.A. Chin, Ann. Phys. (NY) 108 (1977) 301.

[11] P.A.M. Guichon, K. Saito, E. Rodionov, A. W. Thomas, Nucl. Phys. A 601 (1996) 349.

[12] A.L. Licht and A. Pagnamenta, Phys. Rev. D 2 (1970) 1156.

[13] D.H. Lu, A.W. Thomas and A.G. Williams, Phys. Rev. C 55 (1997) 3108.

[14] I.M. Duck, Phys. Lett. B 77 (1978) 223.

[15] S. Théberge and A.W, Thomas, Nucl. Phys. A 393 (1983) 252.

[16] *Review of Particle Physics*, The Eur. Phys. J. C 3 (1998) 1.

[17] X. Jin and B. K. Jennings, Phys. Lett. B 374 (1996) 13; Phys. Rev, C 54 (1996) 1427; H. Müller and B. K. Jennings, Nucl. Phys. A 640 (1998)55.

TABLES

TABLE I. σ and ω coupling constants of the HARTREE+FOCK approximation, for three different free bag radii. Also shown are the ratios of the in-medium to free-space nucleon masses and r.m.s. radii at the saturation density $\rho_0 = 0.15$ fm^{-3}, and the nuclear matter incompressibility K and symmetry energy a_4. The quarks mass is $m_q = 5$ MeV in all cases.

	R (fm)	$g_\sigma^2/4\pi$	$g_\omega^2/4\pi$	M^*/M	r^*/r	K (MeV)	a_4 (MeV)
(a)	0.6	4.36	5.55	0.82	1.01	289	30
(b)	0.8	4.37	5.49	0.83	1.02	285	30
(c)	1.0	4.23	5.03	0.84	1.02	273	30

TABLE II. Contributions of the different mesons to the single-nucleon potentials for $|\boldsymbol{p}| = k_F$ at the saturation density. Columns (a), (b) and (c) correspond to the parameters in Table I. The last line shows the HARTREE values.

Potentials (MeV)		V_s			V_0			U_v		
Parameters		(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)
Direct		-175	-173	-164	$+131$	$+130$	$+119$	–	–	–
Exchange	σ	$+14$	$+12$	$+10$	$+14$	$+12$	$+11$	-0	-0	-0
	ω	-44	-39	-31	$+23$	$+20$	$+16$	-2	-2	-2
	π	-4	-3	-3	-3	-3	-3	-3	-2	-2
	ρ	-32	-34	-32	$+1$	-0	-1	$+8$	$+8$	$+7$
Total		-241	-237	-220	$+166$	$+159$	$+142$	$+3$	$+4$	$+4$
Hartree		-228	-204	-186	$+150$	$+125$	$+108$	–	–	–

FIGURES

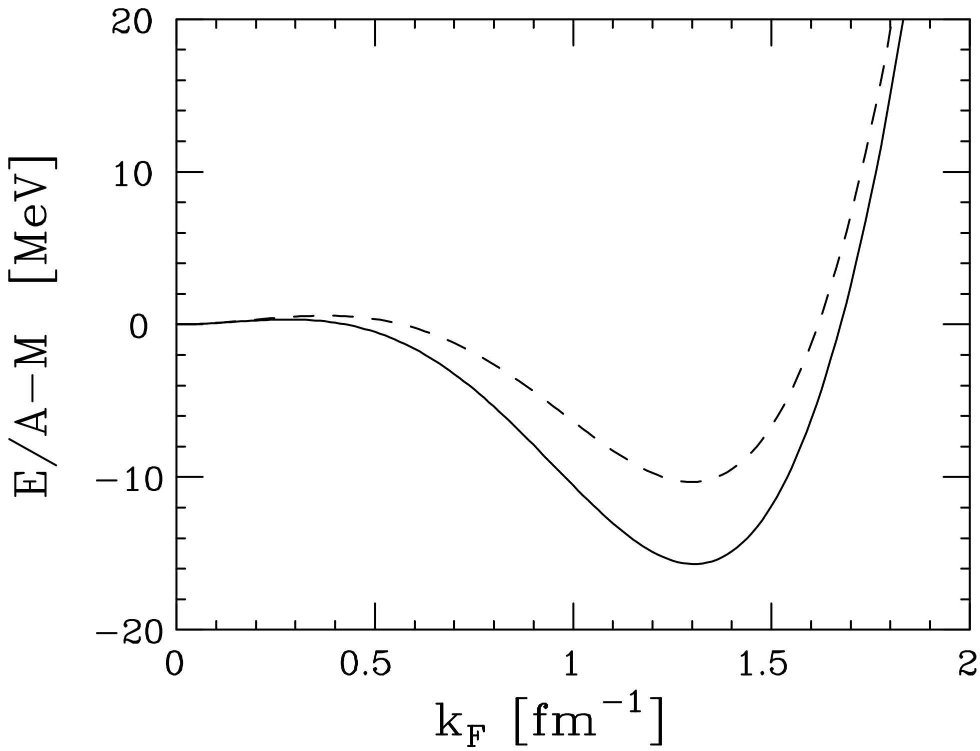

FIG. 1. The energy per particle as a function of the Fermi momentum. The solid curve is the HARTREE result, and the dashed curve is the HARTREE+FOCK result. Here only the σ and ω mesons are included.

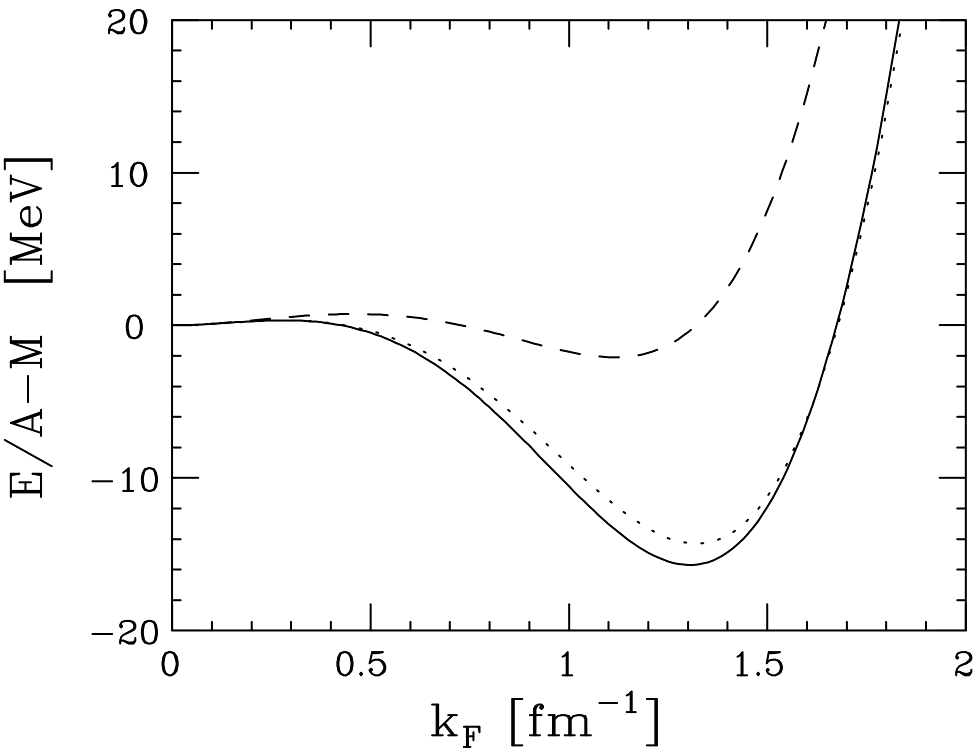

FIG. 2. The energy per particle including the σ, ω and π mesons. The solid curve is the Hartree result, and the dashed (dotted) curve is the HARTREE+FOCK result (contact interaction subtracted).

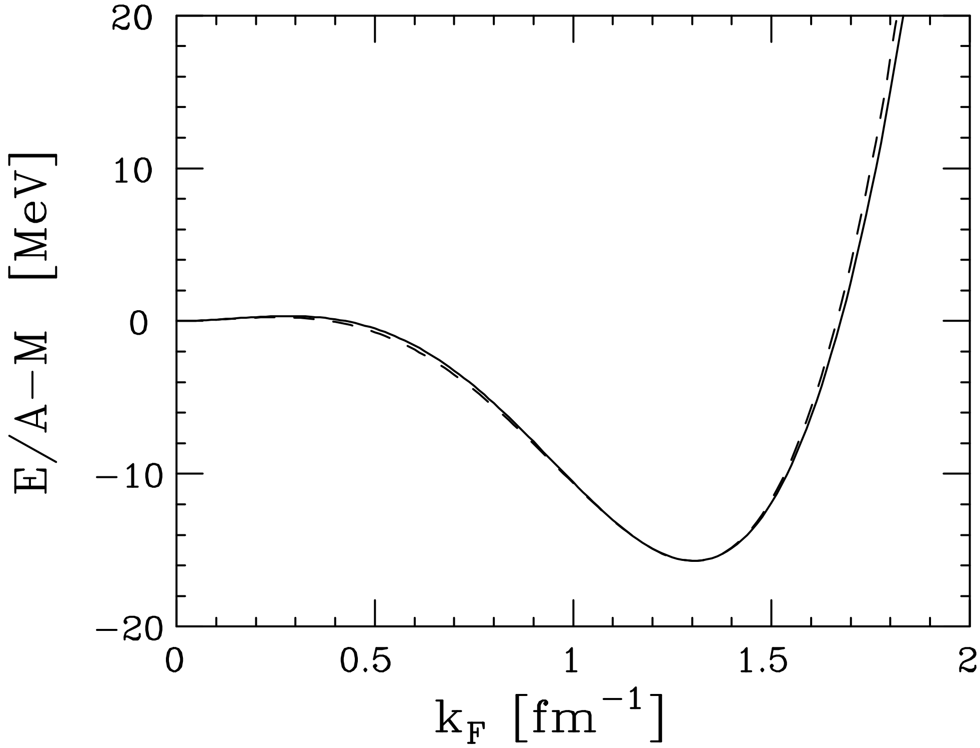

FIG. 3. The energy per particle including the σ, ω, π and ρ mesons. The solid curve is the HARTREE result, and the dashed curve is the HARTREE+FOCK result for the parameter set (b) on Table I. Note that the coupling constants for the HARTREE+FOCK result have been readjusted to produce the same saturation density and energy per nucleon.

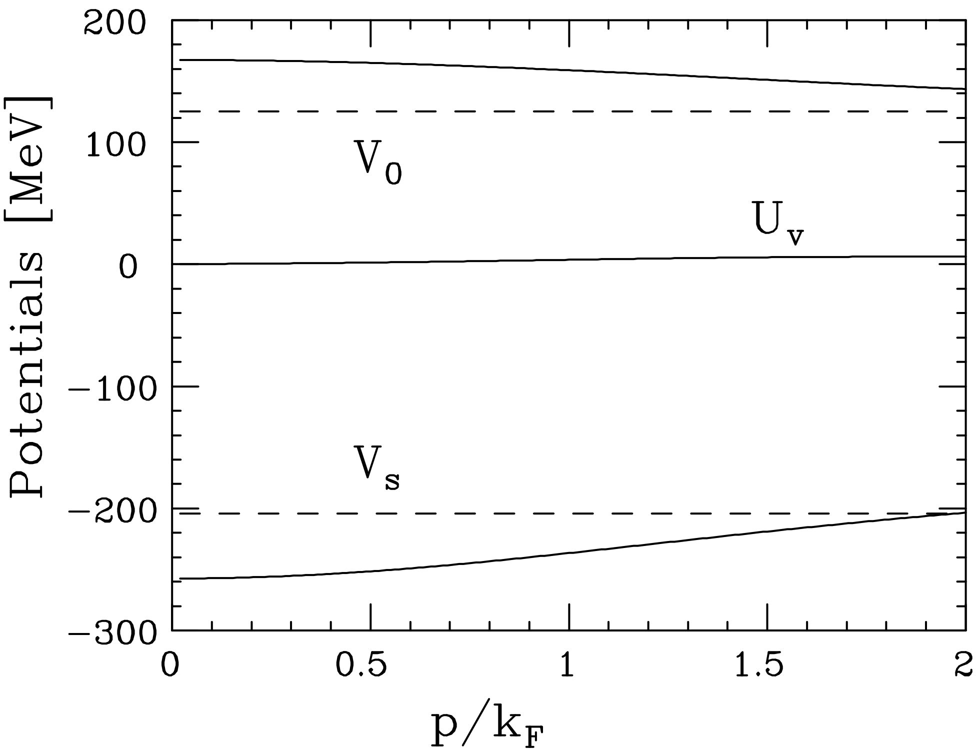

FIG. 4. Single particle potentials $V_s = -g_\sigma\,\sigma_0 + U_s$, $V_0 = g_\omega\,\omega_0 + U_0$, and U_v as a function of p/k_F at the saturation density, $\rho_0 = 0.15\mathrm{fm}^{-3}$. The solid lines are the HARTREE+FOCK results for parameter set (b) in Table I, and the dashed lines are the HARTREE results.